BEAM ENVELOPE OSCILLATIONS AND AMPLITUDE GROWTH IN A HIGH CURRENT COMPACT CYCLOTRON

V. S. Pandit, A. Goswami, P. Sing Babu
Variable Energy Cyclotron Centre, 1- AF, Bidhannagar, Kolkata, India

Abstract

The behaviour of transverse beam oscillations in a compact cyclotron has been discussed for space charge dominated beam using the coupled beam envelope equations. The amplitude growth and oscillations are found to be very sensitive to betatron tunes and space charge effects. We have studied the effect of input beam condition on the amplitude growth and obtained the optimized parameters to get the oscillation amplitude within the acceptable limit.

INTRODUCTION

The results of simulation study in the central region of a 10 MeV compact cyclotron [1,2] reveal that the beam envelope behaves differently due to the coupling of the horizontal and vertical motions arising due to the space charge effect. It is well known that the amount of beam current that can be transported through a smooth focusing channel is a maximum when the beam is perfectly matched to the acceptance of the focusing channel. For a cyclotron with low beam current and without acceleration the matched beam radius is

$$X_m = \sqrt{\frac{\varepsilon_x R}{\nu_r}}, \qquad Z_m = \sqrt{\frac{\varepsilon_z R}{\nu_z}} \tag{1}$$

where ν_r and ν_z are betatron tunes, ε_x and ε_z are beam emittances in horizontal and vertical planes respectively and R is the radius of beam from the machine center. To avoid instabilities the tune value should be chosen properly. In the case of space charge dominated beam together with acceleration where tunes are depressed due to space charge force, the concept of matched beam radius is not well defined. Therefore it is important to study the behaviour of mismatched beam to avoid any beam loss during transport and acceleration.

METHOD OF SIMULATION

In order to obtain the beam envelope $X(s)$ in the horizontal plane and $Z(s)$ in the vertical plane, s being the path length along the accelerated orbit, we have used the following coupled beam envelope equations [3]

$$\frac{d^2X}{ds^2} + \frac{\nu_r^2}{R^2} X - \frac{4I}{(X+Z)I_0\beta^3\gamma^3} \cdot \frac{2\pi}{\Delta\phi} - \frac{\varepsilon_x^2}{X^3} = 0 \tag{2a}$$

$$\frac{d^2Z}{ds^2} + \frac{\nu_z^2}{R^2} Z - \frac{4I}{(X+Z)I_0\beta^3\gamma^3} \cdot \frac{2\pi}{\Delta\phi} - \frac{\varepsilon_z^2}{Z^3} = 0 \tag{2b}$$

The term $(2\pi/\Delta\phi)$ is included with current to account for the phase acceptance in the central region. I_0 = 31 MA for protons. Equations (2) can be solved along the accelerated orbit to obtain X, X' and Z, Z' using the initial values. Since the slope of the envelope reduces with the acceleration, we have modified X' and Z' suitably at each acceleration gap by multiplying them with the ratio of the old $\beta\gamma$ to the new $\beta\gamma$, β and γ being the relativistic terms.

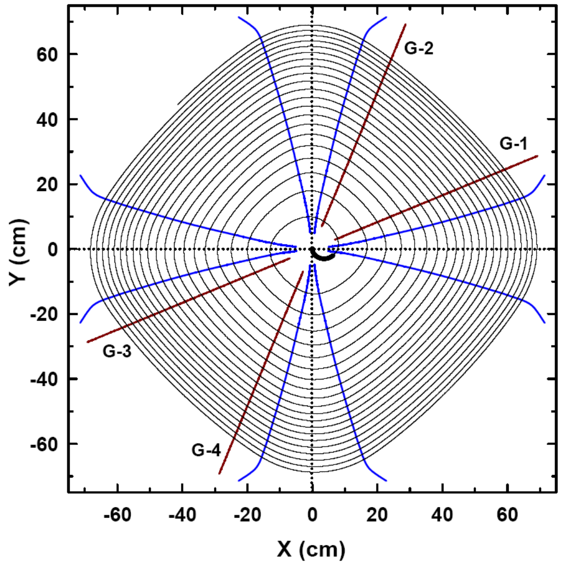

Figure 1: Accelerated orbits for proton from 100 keV to 10 MeV. dee voltage=125 kV, gap width between dee and dummy-dee = 2 cm, dee height = 3 cm.

Fig. 1 shows the horizontal cross section in the central region and position of the inflector, accelerating gaps G-1 to G-4 in the median plane and accelerated orbits of the protons. We have used magnetic field data from 3D MagNet code. The electric field at four gaps of the two resonators in opposite valleys was described by

$$E = \frac{V_D}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{3}$$

with $\sigma = 0.4W + 0.2H$ where H is height of the dee and W is the gap between the dee and the ground electrode. We have solved the coupled differential equations (2) along the accelerated orbits of protons. The radial and vertical betatron tunes ν_r and ν_z were extracted from the data of the magnetic field [4]. The vertical electric focusing at the gaps was determined using the first order theory and included in the vertical tune (see Fig. 2).

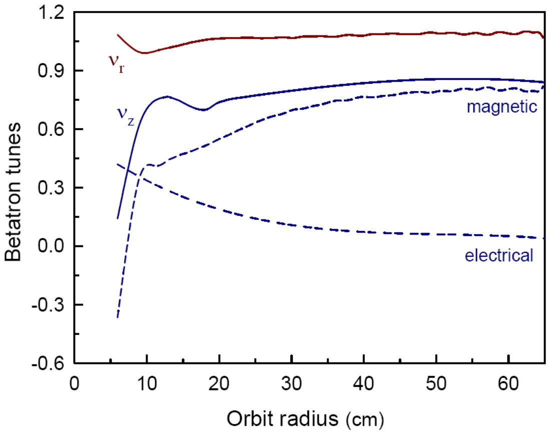

Figure 2: Betatron tunes as a function of orbit radius. Dashed curves represent the contribution to the vertical betatron tunes from the electrical and magnetic focussing.

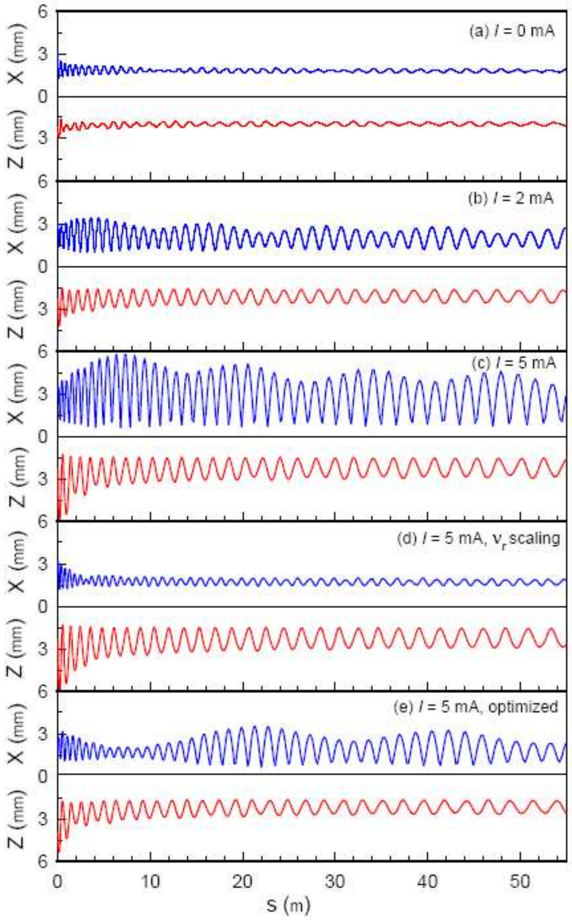

Figure 3: Horizontal (X) and vertical (Z) beam envelopes along the accelerated orbit up to 10 MeV. The initial conditions for plots (a) to (d) are; X_0=3.2mm, X_0'=0mrad, Z_0=2.3mm, Z_0'=0mrad, ε_x=ε_z=60πmmmrad. In case of optimized plot (e) X_0=2mm, X_0'=-15mrad, Z_0=4.1mm, Z_0'=-10mrad, ε_x =42πmmmrad and ε_z =60πmmmrad.

Fig. 3 shows the results of horizontal and vertical beam envelopes tracking along the accelerated orbits up to 21 turns starting from injection radius of 7 cm. We can see from Fig. 3(a) that at low beam intensity with matched initial condition the average amplitude of the beam envelope is limited to 2 mm in both planes. As the beam current increases the transverse space charge effect leads to increase in the beam size and produces envelope oscillations. Fig. 3(b) and 3(c) show the beam envelopes with the same initial conditions as used in Fig. 3(a) for the injected beam current of 2 mA and 5 mA respectively. It is clear that these initial conditions are not at all suitable for injection. We observed large envelope oscillations in the horizontal plane compared to that in the vertical plane. A detailed investigation reveals that amplitude growth and oscillation in the horizontal beam envelope is very sensitive to the radial betatron tune ν_r. The contributions from ellipse orientation, inter plane coupling and space charge effects are very small on these oscillations. A slight change in ν_r reduces the amplitude growth as well as oscillation by large amount as shown in Fig. 3(d) where ν_r is increased by a factor of 1.2 at all radii. We feel that this phenomenon is due to the fact that ν_r is very close to the resonance ν_r = 1. We observed the same result when the value of ν_r is reduced below one. It is not possible to change the profile of ν_r as desired in an isochronous cyclotron because it follows the profile of relativistic term γ as the energy of the beam increases. This value remains close to unity at inner radii where the beam energy is not sufficiently relativistic. The best way to control the beam envelope oscillations and amplitude growth is then to optimize the initial beam parameters i.e. emittance and orientation of the phase ellipse.

Fig. 3(e) shows the optimized beam envelope with different initial conditions. We can see that in this case there is a considerable reduction in the beam size in the radial direction as well as in the amplitude oscillation frequency. The maximum envelope radius in both planes is less than 4 mm for I = 5 mA, which is much less than the chosen maximum height of the dee from the median plane which is equal to 15 mm. Our first order calculations show that it is possible to control 5 mA beam current in the present design of the cyclotron. Here we have presented the preliminary results of our study. A more detailed analysis to dig out the physics behind the envelope oscillations is in progress.

REFERENCES

[1] V. S. Pandit, P. Sing Babu, A. Goswami, P. R. Sarma. 4th Asian Particle Accelerator Conference, APAC-2007, 342.

[2] A. Goswami, P. Sing Babu and V. S. Pandit, Nucl. Instr. and Meth. A 562 (2006) 34.

[3] M. Reiser, “Theory and Design of Charged Particle Beams”, John Wiley and Sons, New York (1994).

[4] P. Sing Babu, A. Goswami, P. R. Sarma and V. S. Pandit, Nucl. Instr. and Meth. A 624 (2010) 560.