HIGHER ORDER MODE ANALYSIS USING PANOFSKY WENZEL THEOREM

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Abstract

Beam cavity interactions can generate higher order electromagnetic modes in an RF cavity. These HOMs can cause beam instabilities and beam loss in the cavities. To analyze the effects of HOMs using Panofsky Wenzel theorem[1], one needs to segregate the TM dipole modes and then use the P-W theorem to calculate the deflecting strength ($R_\perp/Q$). This paper discusses the techniques that are employed to segregate the TM dipole modes and the calculation of their deflecting strength for each of these modes using the P-W theorem. The analysis was used for the HOM study of a proposed design of 31.6 MHz cavity for Indus-1 / Booster Synchrotron.

INTRODUCTION

In the existing scenario, it is difficult to carry out any maintenance activity of the Booster/Indus-I RF cavity without destroying the vacuum condition of the rest of ring. Hence, in order to isolate the cavity from the ring one possible way is the introduction of vacuum bellows at the two sides of the cavity. The main constraint to accommodate these changes is the availability of space. Therefore, to avoid this problem, a reduced length capacitively loaded asymmetric cavity has been proposed where the effect of this reduction in length is compensated by increasing the capacitor plate diameter and reducing the gap(d) between the capacitor plates (Fig. 1). The modified design has been optimised for its performance in TM_{010} mode at a frequency of 31.6 MHz. Thereafter, a Higher Order Modes (HOM) study was taken up using Panofsky-Wenzel Theorem (P-W Theorem). An algorithm was written to segregate the modes and then calculated the ratio of the shunt impedance (R) and Quality factor (Q) for the monopole and dipole modes using the APDL (ANSYS Parametric Design Language).

BACKGROUND

A cylindrical RF cavity can support all different types of TE (having no electric field component parallel to the cylindrical axis) or TM (having no magnetic field component parallel to the cylindrical axis) types of electromagnetic field configurations (modes) satisfying the appropriate boundary conditions[2]. Hence, along with the fundamental one resonating at the lowest frequency, a number of HOMs having resonant frequency less than the cut-off frequency of the beam pipe, are also possible. For TM class cavities, its fundamental mode (TM_{010}) is commonly used to provide acceleration to the beam. While traversing through the cavity the beam bunch can excite a number of HOMs which can interact again to the trailing part of the same bunch or the bunches entering at a later time. Amongst these modes, mainly the monopole and dipole families can influence a well collimated on-axis beam significantly. The number of the HOMs are very much dependent on the cavity geometry and the strength depends on their R/Q. Calculation of this figure of merit ($R_\parallel/Q$) is same as that of an accelerating mode for monopoles but we need to implement, P-W Theorem to calculate the $R_\perp/Q$ for dipole modes. In a cavity, it relates the transverse momentum ($\Delta p_\perp$) of a fast moving particle to the transverse gradient of the axial electric field according to the following expression

$$\Delta p_\perp = j\frac{q}{\omega}\int_{l_1}^{l_2}\nabla_\perp E_z e^{jhz}dz\,, \qquad (1)$$

where l_1 and l_2 are locations, chosen inside the entrance and exit of the cavity beam pipe such that $E(l_1)=E(l_2)=0$, $h=\omega/v$, ω is the resonant frequency for a particular mode and v is the longitudinal velocity of the charged particle. Moving one step ahead, the transverse deflecting voltage $V_\perp$ can be shown as,

$$V_\perp = -\frac{1}{jk}\int_{l_1}^{l_2}\nabla_\perp E_z e^{jhz}dz\,. \qquad (2)$$

Here k is the propagation vector along z direction. Ideally, for the on-axis particles in an accelerating cavity, the deflecting impulse of the electric field in a TE mode exactly cancels the impulse of the magnetic field. Now, from the definition of Shunt Impedance(R), we get $R_{n\perp} = |V_{n\perp}|^2/P_n$ where the suffix n signifies a particular mode and P_n is the power loss on cavity wall for a particular mode[3].

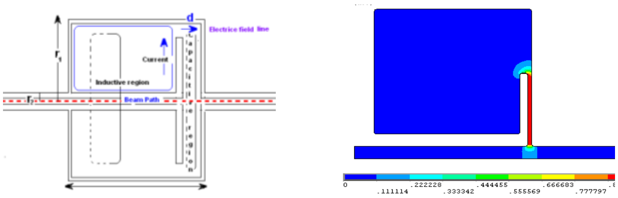

Figure 1: Proposed cavity (left), E-M field distributions.

Simplistic view: Due to introduction of ports and fillet radii at geometrical transitions in a real cavity, the modes no longer remain pure TE or TM but they appear as TE-like and TM-like modes. Still, considering the cylindrically symmetric nature at the accelerating gap, we can use the P-W theorem to calculate the R/Q of the

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HOMs' with the following simplifications:

i] From the above discussion, only the transverse gradient of the *E* field of the TM mode can cause transverse deflection and using cylindrical coordinate system the transverse gradient is given by

$$\nabla_{\perp} E_Z(r,\theta) = \frac{\partial E_Z}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial E_Z}{\partial \theta}\hat{\theta}. \qquad (3)$$

For pure TM monopole mode E_z has a maximum (or minimum) at r=0 and is symmetric in θ. Hence both the terms of the RHS goes to zero. However, for TM dipole modes, $\partial E_z / \partial r \neq 0$ which shows that the 1st term of the RHS has a non zero contribution. So, only the dipole modes can provide nonzero deflection to the on-axis beam (Fig. 2).

ii] Again, for dipole modes, E_z , near axis(r~0), varies almost linearly with respect to *r* Hence, we can write

$$|V_{\perp}| = \frac{1}{|kr_0|}\left|\int_{l_1}^{l_2} E_z(r=r_0)e^{jhz}dz\right|, \qquad (4)$$

where r_0 is small length hence, very near to r=0.

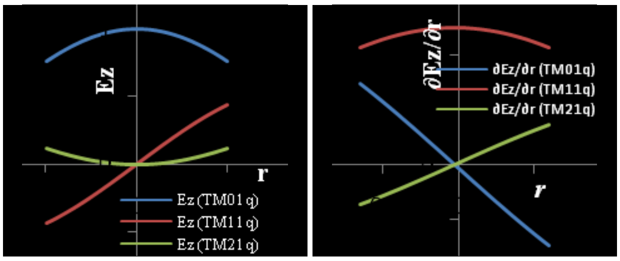

Fig 3: E_z and its derivative for TM monopole and higher order modes in a cylindrical cavity (r~0).

Flow chart for the algorithm: With this basis, the following flow chart for the algorithm was prepared.

1. Set a small value for E_z judiciously (for real cavity TE like modes may also have small $E_z \neq 0$ component) and separate the TM like modes.
2. From the TM like modes select only the monopoles and dipole [(a) For monopoles- take a 5^0 sector model(in θ). Apply 'normal magnetic field (H)' boundary condition at $\theta=0^0$ and $\theta=5^0$. Select the radial location for maximum azimuthal magnetic field (H_θ). If $\partial H_\theta/\partial\theta$ is close to zero in $0^0 \leq \theta \leq 5^0$, then identify the mode as monopole. (b) For dipoles- take a 90^0 sector model (in θ). Apply 'normal magnetic field' boundary condition at $\theta=0^0$ and 'normal electric field' boundary condition at $\theta=90^0$. Check whether $\partial E_z/\partial\theta$ at $r=r_1$ changes its sign or not in between $\theta=0^0$ and 90^0 . If not, identify the mode as dipole).
3. Ensure that the small value $r=r_o$ must be in the regime (near r~0) where E_z varies linearly with r.
4. Perform integration for $|V_{n\perp}|$ and calculate $R_{n\perp}/Q_n$.

RESULTS & DISCUSSIONS

For the proposed design of reduced cavity length (L), simulations were done for two combinations of gap(d) in between the loaded capacitor plate (diameter D) and the end plate. Hence, *R/Q* ,both for monopole and dipole families, were calculated for the two designs.

Design-I: For L=400 mm; D=500 mm; d=11.2 mm;

For the Fundamental mode,

[i] resonant frequency (ω_0)= 32.0641 MHz, [ii] shunt impedance(R)=0.65 MΩ, [iii] Quality factor (Q)=11900,

Design-II: For L=400 mm; D=480 mm; d=10.27 mm;

For the Fundamental mode,

[i] resonant frequency (ω_0)= 32.1068 MHz, [ii] shunt impedance(R)=0.66 MΩ, [iii] Quality factor (Q)=11900,

Table 2: (*R/Q*)Comparison for selected monopoles and dipoles (having ω_0 less than beam pipe cut-off frequency and R/Q more than 1Ω):

Design-I:total number of monopole modes = 36	Design-II :total number of monopole modes = 36
1. 698 MHz -- 11Ω 2. 1146 MHz -- 1.2Ω 3. 1306MHz -- 2 Ω 4. 1359MHz -- 3.9 Ω 5. 1455MHz -- 1.1 Ω 6. 1871MHz -- 1.7 Ω 7. 1957MHz -- 1.7 Ω	1. 722 MHz -- 9.5 Ω 2. 796MHz -- 1.4 Ω 3. 1381MHz -- 4.3 Ω 4. 1461MHz -- 2.2 Ω 5. 1983MHz -- 1.1 Ω 6. 2105MHz -- 1.0 Ω
Design-I :total number of dipole modes = 33	Design-II: Total number of dipole modes = 35
1.for 7 modes R/Q > 1.3Ω 2.for 3 modes R/Q >2.5 Ω 3.for 2 modes R/Q > 5.0Ω	1. for 9 modes R/Q >1.3Ω 2. for 4 modes R/Q > 2.5Ω 3. for 3 modes R/Q > 5.0Ω

Comparing these two set for monopoles the conclusion can be drawn as [i] number of TM_{0nl} modes do not change with reduced gap. [ii] number of TM monopoles having larger *R/Q* decrease as the gap reduces. Similarly for dipoles [i] number of TM_{1nl} modes and their *R/Q* increase as the gap reduces but their magnitude are small.

CONCLUSIONS

We have applied P-W Theorem to study the HOMs on the proposed design. As a concluding remark, we can anticipate that because of this modification the dipole HOMs may become important. Also we have identified particularly dipole modes and have studied their field configurations which will be helpful to design and place the coupler/s to damp the HOMs.

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REFERENCES

[1] W.K.H. Panofsky, W.A. Wenzel, *Rev. Sci. Instr.*,**27**, 967 (1956)

[2] M. J. Browman, *Proc. 1993 Particle Accelerator Conf.* (USA), IEEE, p.800(1993)

[3] Georges Dome, AIP Conf. Pro. No.-153,p1374-1375